$fg^*\alpha$-Continuous Functions In Fuzzy Topological Spaces

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Abstract— This paper deals with several types of fuzzy generalized closed sets and their interrelations. Also $fg^*\alpha$--continuous, $fg^*\alpha$-open functions and $fg^*\alpha$- closed functions are introduced and studied. Again, some important properties of such functions are studied in the newly defined spaces using $fg^*\alpha$–closed sets.

Index Terms—$fg^*\alpha$ –open sets, $fg^*\alpha$ –closed sets, $fg^*\alpha$ –continuity, $fg^*\alpha$ –open functions, $fg^*\alpha$ –closed functions, $fg^*\alpha T_\alpha$ –space, $fg^*\alpha T_c$ space.

1 Introduction

THROUGHOUT the paper, by $(X,\tau),(Y,\tau_1),(Z,\tau_2)$ or simply by X,Y,Z respectively we mean fuzzy topological spaces (fts, for short) in the sense of Chang [3]. A fuzzy set is a mapping from a nonempty set X to the unit closed interval $I=[0,1]$ [6]. $0_X, 1_X$are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement of a fuzzy set A in X will be denoted by $1_X \setminus A$. The two fuzzy sets A and Bin X, we write $A \leq B$if and only if $A(x) \leq B(x)$, for all $x \in X$. $clA$and$intA$ of a fuzzy set A in X [6] respectively stand for the fuzzy closure and fuzzy interior of A in X.

2 $fg^*\alpha$-Open Sets and Its Properties

We now recall the following definitions, which are useful in the sequel.

Definition 2.1. A fuzzy set A in an fts (X,τ) is called fuzzy

(i) semiopen [1] if $A \leq cl\ int\ A$

(ii) α-open [2] if $A \leq int\ cl\ int\ A$

(iii) regular open [1] if $A = int\ cl\ A$

(iv) preopen [5] if $A \leq int\ cl\ A$

The set of all fuzzy semiopen (resp. fuzzy α-open, fuzzy regular open, fuzzy preopen) sets in X is denoted by FSO(X) (resp. FαO(X), FRO(X), FPO(X)).

The complements of the above mentioned sets are called fuzzy semiclosed sets, fuzzy α-closed sets , fuzzy regular closed sets and fuzzy preclosed sets respectively.

Fuzzy semiclosure [1] (resp., fuzzy α-closure [2], fuzzy preclosure [5]) of a fuzzy set Ain X, denoted by $scl\ A$ (resp. $\alpha cl\ A, p\ clA$) is defined to be the intersection of all fuzzy semiclosed (resp., fuzzy α-closed, fuzzy preclosed) sets containing A. It is known that $scl\ A$ (resp. $\alpha cl\ A, pclA$) is a fuzzy semiclosed (resp., fuzzy α-closed, fuzzy preclosed) set.

Definition 2.2. A fuzzy set A in an fts (X,τ) is called fuzzy

(i) generalized closed (fg-closed, for short) if $cl\ A \leq U$ whenever $A \leq U$ and $U \in \tau$,

(ii) semi-generalized closed (fsg-closed, for short) if $scl\ A \leq U$ whenever $A \leq U$ and $U \in$ FSO(X),

(iii) generalized semiclosed (fgs-closed, for short) if $scl\ A \leq U$ whenever $A \leq U$ and $U \in \tau$,

(iv) generalized α-closed ($fg\alpha$-closed, for short) if $\alpha cl\ A \leq U$ whenever $A \leq U$ and $U \in$ FαO(X),

(v) α-generalized closed ($f\alpha g$-closed, for short) if $\alpha cl\ A \leq U$ whenever $A \leq U$ and $U \in \tau$,

(vi) $g^\#$-closed ($fg^\#$-closed, for short) if $cl\ A \leq U$ whenever $A \leq U$ and U is $f\alpha g$-open in (X,τ),

(vii) $wg\alpha$-closed ($fwg\alpha$-closed, for short) if $\alpha cl\ (int\ A) \leq U$ whenever $A \leq U$ and $U \in$ FαO(X),

(viii) $w\alpha g$-closed ($fw\alpha g$-closed, for short) if $\alpha cl\ (int\ A) \leq U$ whenever $A \leq U$ and $U \in \tau$,

(ix) $g^*\alpha$-closed ($fg^*\alpha$-closed, for short) if $\alpha cl\ A \leq U$ whenever $A \leq U$ and U is $fg\alpha$-open in (X,τ),

(x) αgr-closed ($f\alpha gr$-closed, for short) if $\alpha cl\ A \leq U$ whenever $A \leq U$ and $U \in$ FRO(X),

(xi) gpr-closed ($fgpr$-closed, for short) if $pcl\ A \leq U$ whenever $A \leq U$ and $U \in$FRO(X).

The complements of the above mentioned sets are called their respective open sets.

Definition 2.3. An fts(X,τ) is called an

(i) fT_b-space if every fgs-closed set in (X,τ) is fuzzy closed in (X,τ),

(ii) $f\alpha T_b$-space if every $f\alpha g$-closed set in (X,τ) is fuzzy closed in (X,τ),

(iii) $fg^*\alpha T_c$-space if every $fg^*\alpha$-closed set in (X,τ) is fuzzy closed in (X,τ),

(iv) $fg^*\alpha T_\alpha$-space if every $fg^*\alpha$-closed set in (X,τ) is fuzzy α-closed in (X,τ),

(v) $fwg\alpha T_{g^*\alpha}$-space if every $fwg\alpha$-closed set n (X,τ) is $fg^*\alpha$-closed in (X,τ).

Definition 2.4. A function $f:(X,\tau)\to(Y,\tau_1)$ is called fuzzy

(i) α-continuous [4] ($f\alpha$-continuous, for short) if $f^{-1}(V) \in$ FαO(X) for every $V \in \tau_1$,

(ii) semicontinuous [1] (fs-continuous, for short) if $f^{-1}(V) \in$ FSO(X) for every $V \in \tau_1$,

(iii) g-continuous (fg-continuous, for short) if $f^{-1}(V)$ is fuzzy g-open in (X,τ) for every $V \in \tau_1$,

(iv) sg-continuous (fsg-continuous, for short) if $f^{-1}(V)$ is fsg-open in $(X,\ \tau)$ for every $V \in \tau_1$,

(v) gs-continuous (fgs-continuous, for short) if

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$f^{-1}(V)$ is fgs-open in (X,τ) for every $V \in \tau_1$,

(vi) $g\alpha$-continuous ($fg\alpha$-continuous, for short) if $f^{-1}(V)$ is $fg\alpha$-open in (X,τ) for every $V \in \tau_1$,

(vii) αg-continuous ($f\alpha g$-continuous, for short) if $f^{-1}(V)$ is $f\alpha g$-open in (X,τ) for every $V \in \tau_1$,

(viii) Completely continuous if $f^{-1}(V) \in$ FRO(X) for every $V \in \tau_1$,

(ix) α-irresolute ($f\alpha$-irresolute, for short) if $f^{-1}(V) \in$ FαO(X) for every $V \in$ FαO(Y),

(x) $wg\alpha$-continuous ($fwg\alpha$-continuous, for short) if $f^{-1}(V)$ is $fwg\alpha$-open in (X,τ) for every $V \in \tau_1$,

(xi) $w\alpha g$-continuous ($fw\alpha g$-continuous, for short) if $f^{-1}(V)$ is $fw\alpha g$-open in (X,τ) for every $V \in \tau_1$,

(xii) $g^{\#}$-continuous ($fg^{\#}$-continuous, for short) if $f^{-1}(V)$ is $fg^{\#}$-open in (X,τ) for every $V \in \tau_1$,

(xiii) gpr-continuous ($fgpr$-continuous, for short) if $f^{-1}(V)$ is $fgpr$-open in (X,τ) for every $V \in \tau_1$,

(xiv) αgr-continuous ($f\alpha gr$-continuous, for short) if $f^{-1}(V)$ is $f\alpha gr$-open in (X,τ) for every $V \in \tau_1$.

Proposition 2.5. *Every fuzzy open set V in an fts (X,τ) is $fg^*\alpha$-open in (X,τ).*

Proof. Let $V \in \tau$ be arbitrary. Then $1_X\backslash V \in \tau^c$. Let $1_X\backslash V \le G$ where G is $fg\alpha$-open in (X,τ). Then $\alpha cl\,(1_X\backslash V) \le cl\,(1_X\backslash V) = 1_X\backslash V \le G$. Therefore, $1_X\backslash V$ is $fg^*\alpha$-closed in (X,τ) and hence $V$is $fg^*\alpha$-open in (X,τ).

Proposition 2.6.*Every fuzzy regular open set in an fts(X,τ) is $fg^*\alpha$-open in (X,τ).*

Proof. Since every fuzzy regular open set is fuzzy open, the proof follows from Proposition 2.5.

Proposition 2.7.*Every $fg^*\alpha$-open set in an fts(X,τ) is $fg\alpha$-open in (X,τ).*

Proof. Let V be $fg^*\alpha$-open set in (X,τ). Then $1_X\backslash V$ is $fg^*\alpha$-closed in (X,τ). Let $U \in$ FαO(X) be such that $1_X\backslash V \le U$. Then U is fuzzy $fg\alpha$-open in (X,τ). Indeed, $1_X\backslash U$ is fuzzy α-closed in (X,τ) and let $1_X\backslash U \le W$ where $W \in$ FαO(X). Then $\alpha cl\,(1_X\backslash U) = 1_X\backslash U \le W$ and so $1_X\backslash U$ is $fg\alpha$-closed and hence U is $fg\alpha$-open in (X,τ). Since $1_X\backslash$V is $fg^*\alpha$-closed in (X,τ) and $1_X\backslash V \le U$ where U is $fg\alpha$-open in (X,τ), $\alpha cl\,(1_X\backslash V) \le U$ implies that $1_X\backslash V$ is $fg\alpha$-closed and hence V is $fg\alpha$-open in (X,τ).

Proposition 2.8.*Every fuzzy a-open set is $fg^*\alpha$-open set in (X,τ).*

Proof. Let $U \in$ FαO(X). Then $1_X\backslash U$ is fuzzy α-closed in (X,τ). Let $1_X\backslash U \le G$ where G is $fg\alpha$-open in (X,τ). Then $\alpha cl\,(1_X\backslash U) = 1_X\backslash U \le G$ and so $1_X\backslash U$ is $fg^*\alpha$-closed set and hence U is $fg^*\alpha$-open set in (X,τ).

Proposition 2.9.*Every $fg^*\alpha$-open set is $f\alpha g$-open n (X,τ).*

Proof. Let U be an $fg^*\alpha$-open set in (X,τ). Then $1_X\backslash U$ is $fg^*\alpha$-closed in (X,τ). Let $V \in \tau$ be such that $1_X\backslash U \le V$. Then $V \in$ FαO(X) and so by Proposition 2.8, V is $fg^*\alpha$-open and hence by Proposition 2.7, V is $fg\alpha$-open in (X,τ). Since $1_X\backslash U$ is $fg^*\alpha$-closed, $\alpha cl\,(1_X\backslash U) \le V$ and then $1_X\backslash U$ is $f\alpha g$-closed and consequently, U is $f\alpha g$-open in (X,τ).

Proposition 2.10. *Every $fg^{\#}$-open set is $fg^*\alpha$-open in (X,τ).*

Proof. Let Abe $fg^{\#}$-open set in an fts (X,τ). Then $1_X\backslash A$ is $fg^{\#}$-closed in (X,τ). Let G be any $fg\alpha$-open set in X such that $1_X\backslash A \le G$. Then G is $f\alpha g$-open set in X. Indeed, $1_X\backslash G$ is $fg\alpha$-closed in X. Let $W \in \tau$ be such that $1_X\backslash$G$\le W$. Then $W \in$ FαO(X). Then $\alpha cl\,(1_X\backslash$G$) \le W$ and so $1_X\backslash G$ is $f\alpha g$-closed and hence G is $f\alpha g$-open in (X,τ). Therefore, $cl\,(1_X\backslash A) \le G \implies \alpha cl\,(1_X\backslash A) \le cl\,(1_X\backslash A) \le G$ and so $1_X\backslash A$ is $fg^*\alpha-$closed and hence A is $fg^*\alpha$-open in (X,τ).

Proposition 2.11.*Every $fg^*\alpha$-open set is $fwg\alpha$-open in (X,τ).*

Proof. Let U be $fg^*\alpha$-open in (X,τ). Then $1_X\backslash U$ is $fg^*\alpha$-closed in (X,τ). Let $G \in$ FαO(X) be such that $1_X\backslash U \le G$. Then by Proposition 2.7 and Proposition 2.8, G is $fg\alpha$-open in (X,τ). Since $1_X\backslash U$ is $fg^*\alpha$-closed, $\alpha cl\,(1_X\backslash U) \le G \implies \alpha cl\, int\,(1_X\backslash U) \le \alpha cl\,(1_X\backslash U) \le G$ and so $1_X\backslash U$is $fwg\alpha$-closed and hence U is $fwg\alpha$-open in (X,τ).

Proposition 2.12.*Every $fg^*\alpha$-open set is fgs-open set in (X,τ).*

Proof. Let U be $fg^*\alpha$-open in (X,τ). Then $1_X\backslash U$ is $fg^*\alpha$-closed in X. Let $V \in \tau$ be such that $1_X\backslash U \le V$. Then $V \in$ FαO(X) and then by Proposition 2.7 and Proposition 2.8, V is $fg\alpha$-open set in X. As $1_X\backslash U$ is $fg^*\alpha$-closed, $\alpha cl\,(1_X\backslash U) \le V \implies scl\,(1_X\backslash U) \le \alpha cl\,(1_X\backslash U) \le V$ and so $1_X\backslash U$ is fgs-closed in X and consequently, U is fgs-open in X.

Proposition 2.13.*Every $fg^*\alpha$-open set is $f\alpha gr$-open in (X,τ).*

Proof. Let A be $fg^*\alpha$-open in X. Then $1_X\backslash A$ is $fg^*\alpha$-closed in X. Let $U \in$FRO(X) be such that $1_X\backslash A \le U$. Since $U \in$FRO(X) $\Rightarrow U \in \tau$ and hence$U$ is $fg\alpha$-open in X, as $1_X\backslash A$ is $fg^*\alpha$-closed, $\alpha cl\,(1_X\backslash A) \le U$ and hence $1_X\backslash A$ is $f\alpha gr$-closed in X and consequently, A is $f\alpha gr$-open in (X,τ).

Proposition 2.14.*Every $fg\alpha$-open set is $f\alpha g$-open set in (X,τ).*

Proof. Let A be $fg\alpha$-open in X. Then $1_X\backslash A$ is $fg\alpha$-closed in X. Let $U \in \tau$ be such that $1_X\backslash A \le U$. Then $U \in$ FαO(X) and so $\alpha cl\,(1_X\backslash A) \le U$ and so $1_X\backslash A$ is $f\alpha g$-closed and hence A is $f\alpha g$-open in X.

Proposition 2.15.*Let $f : (X,\tau) \to (Y,\tau_1)$ be a fuzzy function. Then the following statements are true :*

(i) $f$is fuzzy continuous [3] implies f is $fg\alpha$-continuous.

(ii) $f$is$fwg\alpha$-continuous implies f is $fw\alpha g$-continuous.

(iii) *$f$is$f\alpha gr$-continuous implies f is $fgpr$-continuous.*

Proof. (i) Let f be fuzzy continuous and $V \in \tau_1$. Then $f^{-1}(V) \in \tau$. Since every fuzzy open set is $fg\alpha$-open in X (by Proposition 2.5 and Proposition 2.7), $f^{-1}(V)$ is $fg\alpha$-open in X

and hence f is $fg\alpha$-continuous.
(ii) Let $f$be $fwg\alpha$-continuous and $V \in \tau_1$. Then $f^{-1}(V)$ is $fw\alpha g$-open in$X$. We claim that $f^{-1}(V)$ is $fwg\alpha$-open in X. Indeed, let U be any $fwg\alpha$-open in X. Then $1_X\backslash U$ is $fwg\alpha$-closed in$X$. Let $G \in \tau$ be such that $1_X\backslash U \leq G$. Then $G \in$ FαO(X) and as $1_X\backslash U$ is $fwg\alpha$-closed, $\alpha cl\left(int\,(1_X\backslash U)\right) \leq G$ and so $1_X\backslash U$ is $fw\alpha g$-closed and hence U is $fw\alpha g$-open in X. Hence f is $fw\alpha g$-continuous.
(iii) Let f be $f\alpha gr$-continuous and$V \in \tau_1$. Then $f^{-1}(V)$ is $f\alpha gr$-open in (X,τ). Since fuzzy α-open sets are fuzzy preopen, it follows that for any $A \in I^X$, $pcl\,A \leq \alpha cl\,A$ and hence f is $fgpr$-continuous.

Definition 2.16. A fuzzy function $f:(X,\tau) \to (Y,\tau_1)$ is called fuzzy pre-α-closed if $f(\alpha cl\,A)$ is fuzzy α-closed in (Y,τ_1), for every fuzzy set A in X.

3 $fg^*\alpha$-CONTINUOUS FUNCTIONS

In this section the concept of $fg^*\alpha$-continuous function in an fts (X,τ) has been introduced and studied some of its properties and found the relationship of this function with the previously defined functions.

Definition 3.1. A fuzzy function $f:(X,\tau) \to (Y,\tau_1)$ is said to be fuzzy generalized*α-continuous ($fg^*\alpha$ –continuous, for short) if$f^{-1}(V)$is $fg^*\alpha$-open in X for every $V \in \tau_1$.

Theorem 3.2.*Every fuzzy continuous function $f:(X,\tau) \to (Y,\tau_1)$ is $fg^*\alpha$-continuous.*

Proof. Let $V \in \tau_1$. Then $f^{-1}(V) \in \tau$. By Proposition 2.5, $f^{-1}(V)$ is $fg^*\alpha$-open in X and hence f is $fg^*\alpha$-continuous.

Remark 3.3. The converse of the above theorem need not be true as seen from the following example.

Example 3.4. *$fg^*\alpha$-continuity $\nRightarrow$ fuzzy continuity*
Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A\}$, $\tau_1 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.4, B(b) = 0.4$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau) \to (X,\tau_1)$. Now $i^{-1}(1_X\backslash B) = 1_X\backslash B$ and 1_X is the only $fg\alpha$-open set in (X,τ) containing $1_X\backslash B$ and so i is $fg^*\alpha$-continuous. Again, $B \in \tau_1$and $i^{-1}(B) = B \notin \tau_1$. Hence i is not fuzzy continuous.

Theorem 3.5.*Every fuzzy completely continuous function is $fg^*\alpha$-continuous.*

Proof. Let$f:(X,\tau) \to (Y,\tau_1)$ be fuzzy completely continuous function and $V \in \tau_1$ be arbitrary. Then $f^{-1}(V) \in$FRO(X) and hence $f^{-1}(V) \in \tau$ and then by Proposition 2.5, $f^{-1}(V)$ is $fg^*\alpha$-open in X. Consequently, f is $fg^*\alpha$-continuous.

Remark 3.6. The converse of the above theorem need not be true in general as seen from the following example.

Example 3.7.*$fg^*\alpha$-continuity $\nRightarrow$ fuzzy completely continuity*
Consider Example 3.4. Here i is $fg^*\alpha$-continuous. Now $B \in \tau_1$ and $i^{-1}(B) = B \notin$ FRO(X,τ). Hence i is not fuzzy completely continuous.

Theorem 3.8.*Every $fg^*\alpha$-continuous function is$fg\alpha$-continuous.*

Proof. Let $f:(X,\tau) \to (Y,\tau_1)$ be $fg^*\alpha$-continuous and $V \in \tau_1$. Then $f^{-1}(V)$ is $fg^*\alpha$-open in X. By Proposition 2.7, $f^{-1}(V)$ is $fg\alpha$-open in X and hence f is $fg\alpha$-continuous.

Remark 3.9. The converse of the above theorem need not be true as seen from the following example.

Example 3.10.*$fg\alpha$-continuity$\nRightarrow$ $fg^*\alpha$-continuity*
Let $X = \{a,b\}$, $\tau = \{0_X, 1_X, A\}$, $\tau_1 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.6, B(b) = 0.4$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau) \to (X,\tau_1)$. Fuzzy α-open sets in $(X,\tau)$are $0_X, 1_X, A$. Then fuzzy α-closed sets in $(X,\tau)$are $0_X, 1_X, 1_X\backslash A$. Now $fg\alpha$-closed sets in (X,τ) are $0_X, 1_X, U$ where $U \nleq A$ and so $fg\alpha$-open sets in (X,τ) are $0_X, 1_X, 1_X\backslash U$where $1_X\backslash U \ngeq 1_X\backslash A$. Now $1_X\backslash B \in \tau_1^c$.$i^{-1}(1_X\backslash B) = 1_X\backslash B$which is $fg\alpha$-closed in (X,τ). Therefore, i is $fg\alpha$-continuous. But $1_X\backslash B$ is not $fg^*\alpha$-closed as $1_X\backslash B$ is $fg\alpha$-open in $(X,\tau)$and $\alpha cl\,(1_X\backslash B) = 1_X\backslash A \nleq 1_X\backslash B$. Hence i is not $fg^*\alpha$-continuous.

Theorem 3.11. *Every $f\alpha$-continuous function is $fg^*\alpha$-continuous.*

Proof. Let $f:(X,\tau) \to (Y,\tau_1)$ be $f\alpha$-continuous and $V \in \tau_1$. Then $f^{-1}(V) \in$ FαO(X). By Proposition 2.8, $f^{-1}(V)$is $fg^*\alpha$-open in (X,τ) and hence f is $fg^*\alpha$-continuous.

Remark 3.12. The converse of the above theorem need not be true as seen from the following example.

Example 3.13.*$fg^*\alpha$-continuity$\nRightarrow$ $f\alpha$-continuity*
Consider Example 3.4. Here i is $fg^*\alpha$-continuous. Now $B \in \tau_1$, $i^{-1}(B) = B \notin$ FαO(X,τ). Hence i is not $f\alpha$-continuous.

Theorem 3.14.*Every $fg^*\alpha$-continuous function is $f\alpha g$-continuous.*

Proof. Let $f:(X,\tau) \to (Y,\tau_1)$be $fg^*\alpha$-continuous and $V \in \tau_1$. Then $f^{-1}(V)$ is $fg^*\alpha$-open in X. By Proposition 2.9, $f^{-1}(V)$ is $f\alpha g$-open in X and hence f is $f\alpha g$-continuous.

Remark 3.15. The converse of the above theorem need not be true as seen from the following example.

Example 3.16.*$f\alpha g$-continuity$\nRightarrow$ $fg^*\alpha$-continuity*
Let $X = \{a\}$, $\tau = \{0_X, 1_X, B\}$, $\tau_1 = \{0_X, 1_X, A\}$ where $B(a) = 0.6$ and $A(a) = \frac{1}{3}$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau) \to (X,\tau_1)$. We claim that i is $f\alpha g$-continuous but not $fg^*\alpha$-continuous.
Now fuzzy α-open sets in (X,τ) are $0_X, 1_X, B, U$where $U(a) \geq 0.6$. Then fuzzy α-closed sets in (X,τ) are $0_X, 1_X, 1_X\backslash B, 1_X\backslash U$where $(1_X\backslash B)(a) = 0.4, (1_X\backslash U)(a) \leq 0.4$. Again $fg\alpha$-closed sets in (X,τ) are $0_X, 1_X, V$ where $V(a) \leq 0.4$ [Indeed, $\alpha cl\,V \leq 1_X\backslash B$whereas $V \leq U$]. And so $fg\alpha$-open sets in (X,τ) are $0_X, 1_X, 1_X\backslash V$ where $(1_X\backslash V)(a) \geq 0.6$.Now $1_X\backslash A \in \tau_1^c$.

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Therefore, $i^{-1}(1_X\backslash A) = 1_X\backslash A$ is $fg\alpha$-open set in (X,τ). Therefore, $1_X\backslash A \leq 1_X\backslash A$, but $\alpha cl\,(1_X\backslash A) = 1_X \nleq 1_X\backslash A$. Therefore, $1_X\backslash A$ is not $fg^*\alpha$-closed in (X,τ) and so i is not $fg^*\alpha$-continuous. Again, 1_X is the only fuzzy open set in (X,τ) such that $1_X\backslash A \leq 1_X$.

Proposition 3.17. *Let $f:(X,\tau)\to(Y,\tau_1)$ be an $f\alpha g$-continuous function where (X,τ) is an $f\alpha T_b$-space. Then f is $fg^*\alpha$-continuous.*

Proof. Let $V \in \tau_1$. As f is $f\alpha g$-continuous, $f^{-1}(V)$ is $f\alpha g$-open in (X,τ). Then $1_X\backslash f^{-1}(V)$ is $f\alpha g$-closed in (X,τ). As (X,τ) is $f\alpha T_b$-space, $1_X\backslash f^{-1}(V)$ is fuzzy closed in (X,τ) and hence $f^{-1}(V)$ is fuzzy open in (X,τ) . By Proposition 2.5, $f^{-1}(V)$ is $fg^*\alpha$-open in (X,τ) and hence f is $fg^*\alpha$-continuous.

Theorem 3.18. *Every $fg^{\#}$-continuous function is $fg^*\alpha$-continuous.*

Proof. Let $V \in \tau_1$. Then $f^{-1}(V)$ is $fg^{\#}$-open in (X,τ). By Proposition 2.10, $f^{-1}(V)$ is $fg^*\alpha$-open in (X,τ) and hence f is $fg^*\alpha$-continuous.

Remark 3.19. The converse of the above theorem need not be true as seen from the following example.

Example 3.20. *$fg^*\alpha$-continuity $\nRightarrow$ $fg^{\#}$- continuity*

Let $X=\{a,b\}$, $\tau=\{0_X,1_X,A\}$, $\tau_1=\{0_X,1_X,B\}$ where $A(a)=0.4, A(b)=0.6, B(a)=0.5, B(b)=0.7$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau)\to(X,\tau_1)$. Now fuzzy α-open sets in (X,τ) are $0_X,1_X,A,U$ where $U\geq A$ and so fuzzy α-closed sets in (X,τ) are $0_X,1_X,1_X\backslash A,1_X\backslash U$ where $1_X\backslash U \leq 1_X\backslash A$. Now $fg\alpha$-closed sets in (X,τ) are $0_X,1_X,1_X\backslash A,1_X\backslash U$ and so $fg\alpha$-open sets in (X,τ) are $0_X,1_X,A,U$. Again, $f\alpha g$-closed sets in (X,τ) are $0_X,1_X,V,W$ where $V(a)\leq 0.4, V(b)\leq 0.4$ and $W>A$. Then $f\alpha g$-open sets in (X,τ) are $0_X,1_X,1_X\backslash V,1_X\backslash W$ where $1-V(a)\geq 0.6, 1-V(b)\geq 0.6$ and $1_X\backslash W<1_X\backslash A$.
Now $1_X\backslash B \in \tau_1^c$ and $i^{-1}(1_X\backslash B) = 1_X\backslash B$ which is $f\alpha g$-open set in (X,τ). But $cl_\tau(1_X\backslash B) = 1_X\backslash A \nleq 1_X\backslash B$. Therefore, i is not $fg^{\#}$-continuous. Again, $U(a)\geq 0.5, U(b)\geq 0.6$ are $fg\alpha$-open sets in (X,τ) containing $1_X\backslash B$ and $\alpha cl_\tau(1_X\backslash B) = 1_X\backslash B \leq U$. Hence i is $fg^*\alpha$-continuous.

Theorem 3.21. *Every $fg^*\alpha$-continuous function is $fwg\alpha$-continuous.*

Proof. Let $f:(X,\tau)\to(Y,\tau_1)$ be $fg^*\alpha$-continuous and $V\in\tau_1$. Then $f^{-1}(V)$ is $fg^*\alpha$-open in X. By Proposition 2.11, $f^{-1}(V)$ is $fwg\alpha$-open in X and hence f is $fwg\alpha$-continuous.

Remark 3.22. The converse of the above theorem need not be true as seen from the following example.

Example 3.23. $fwg\alpha$-continuity $\nRightarrow$ *$fg^*\alpha$-continuity*

Consider Example 3.10. Here $1_X\backslash B$ is $fwg\alpha$-closed as 1_X is the only fuzzy α-open set in (X,τ) containing $1_X\backslash B$.

Proposition 3.24. *Let $f:(X,\tau)\to(Y,\tau_1)$ be an $fwg\alpha$-continuous function where (X,τ) is an $fwg\alpha T_{g^*\alpha}$-space. Then f is $fg^*\alpha$-continuous.*

Proof. Let $V\in\tau_1$. As f is $fwg\alpha$-continuous, $f^{-1}(V)$ is $fwg\alpha$-open in (X,τ). As (X,τ) is $fwg\alpha T_{g^*\alpha}$-space, $1_X\backslash f^{-1}(V)$ is $fg^*\alpha$-closed in (X,τ) and hence $f^{-1}(V)$ is $fg^*\alpha$-open in (X,τ). Consequently, f is $fg^*\alpha$-continuous.

Theorem 3.25. *Every $fg^*\alpha$-continuous function is $fw\alpha g$-continuous.*

Proof. Let $f:(X,\tau)\to(Y,\tau_1)$ be $fg^*\alpha$-continuous function. By Theorem 3.21, f is $fwg\alpha$-continuous. Then by Proposition 2.15(ii), f is $fw\alpha g$-continuous.

Remark 3.26. The converse of the above theorem need not be true as seen from the following example.

Example 3.27. *$fw\alpha g$-continuity $\nRightarrow$ $fg^*\alpha$-continuity*

Consider Example 3.16. Here $1_X\backslash A\in\tau_1^c$, $i^{-1}(1_X\backslash A) = 1_X\backslash A$. $1_X\backslash A \leq 1_X$ where 1_X is the only fuzzy open set in (X,τ). Now, $\alpha cl_\tau\big(int_\tau(1_X\backslash A)\big) = \alpha cl_\tau B = 1_X \leq 1_X$. Therefore, $1_X\backslash A$ is $fw\alpha g$-closed in (X,τ) and hence i is $fw\alpha g$-continuous though it is not $fg^*\alpha$-continuous.

Theorem 3.28. *Every $fg^*\alpha$-continuous function is fgs-continuous.*

Proof. Let $f:(X,\tau)\to(Y,\tau_1)$ be $fg^*\alpha$-continuous and $V\in\tau_1$. As f is $fg^*\alpha$-continuous, $f^{-1}(V)$ is $fg^*\alpha$-open in (X,τ). By Proposition 2.12, $f^{-1}(V)$ is fgs-open in (X,τ) and hence f is fgs-continuous.

Remark 3.29. The converse of the above theorem need not be true as seen from the following example.

Example 3.30. *fgs-continuity $\nRightarrow$ $fg^*\alpha$-continuity*

Consider Example 3.16. Since 1_X is the only fuzzy open set in (X,τ) such that $1_X\backslash A \leq 1_X$, $scl_\tau(1_X\backslash A)\leq 1_X$ and hence $1_X\backslash A$ is fgs-closed set in (X,τ). Hence i is fgs-continuous.

Proposition 3.31. *Let $f:(X,\tau)\to(Y,\tau_1)$ be an fgs-continuous function where (X,τ) is an fT_b-space. Then f is $fg^*\alpha$-continuous.*

Proof. Let $V\in\tau_1$. As f is fgs-continuous, $f^{-1}(V)$ is fgs-open in (X,τ). Then $1_X\backslash f^{-1}(V)$ is fuzzy closed in (X,τ). Hence ,$f^{-1}(V)$ is fuzzy open in (X,τ). By Proposition 2.5, $f^{-1}(V)$ is $fg^*\alpha$-open in (X,τ) and hence f is $fg^*\alpha$-continuous.

Theorem 3.32. *Every $fg^*\alpha$-continuous function is $f\alpha gr$-continuous.*

Proof. Let $f:(X,\tau)\to(Y,\tau_1)$ be $fg^*\alpha$-continuous and $V\in\tau_1$. Then $f^{-1}(V)$ is $fg^*\alpha$-open in (X,τ). By Proposition 2.13, $f^{-1}(V)$ is $f\alpha gr$-open in (X,τ). Hence f is $f\alpha gr$-continuous.

Remark 3.33. The converse of the above theorem need not be true as seen from the following example.

Example 3.34. *$f\alpha gr$-continuity $\nRightarrow$ $fg^*\alpha$-continuity*

Consider Example 3.16. The only fuzzy regular open sets in

$(X,\tau)$are $0_X, 1_X$. Therefore, $1_X\backslash A \le 1_X \Rightarrow \alpha cl_\tau(1_X\backslash A) = 1_X \le 1_X \Rightarrow 1_X\backslash A$ is $f\alpha gr$-closed in (X,τ). Hence i is $f\alpha gr$-continuous though it is not $fg^*\alpha$-continuous.

Theorem 3.35. *Every $fg^*\alpha$-continuous function is $fgpr$-continuous.*

Proof. By Theorem 3.32, every $fg^*\alpha$-continuous function is $f\alpha gr$-continuous and again by Proposition 2.5(iii), it is $fgpr$-continuous.

Remark 3.36. The converse of the above theorem need not be true as seen from the following example.

Example 3.37. *$fgpr$-continuity$\nRightarrow$ $fg^*\alpha$-continuity*
Consider Example 3.16. The only fuzzy regular open setss in $(X,\tau)$are $0_X, 1_X$. Now$1_X\backslash A \le 1_X \Rightarrow pcl_\tau(1_X\backslash A) = 1_X \le 1_X \Rightarrow 1_X\backslash A$ is $fgpr$-closed in (X,τ) and hence i is $fgpr$-continuous though it is not $fg^*\alpha$-continuous.

Theorem 3.38. *If a fuzzy function $f:(X,\tau)\to(Y,\tau_1)$ is $f\alpha$-irresolute, then it is $fg^*\alpha$-continuous.*

Proof. Let $V \in \tau_1$. Then $V \in$ FαO(Y). As f is $f\alpha$-irresolute, $f^{-1}(V) \in$ FαO(X). By Proposition 2.8, $f^{-1}(V)$ is $fg^*\alpha$-open in (X,τ) and hence f is $fg^*\alpha$-continuous.

Remark 3.39. The converse of the above theorem need not be true as seen from the following example.

Example 3.40. *$fg^*\alpha$-continuity$\nRightarrow$ $f\alpha$- continuity*
Let $X=\{a,b\}$, $\tau=\{0_X,1_X,A\}$, $\tau_1=\{0_X,1_X,B\}$ where $A(a)=0.5, A(b)=0.4, B(a)=0.4, B(b)=0.4$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau)\to(X,\tau_1)$. Now $i^{-1}(1_X\backslash B) = 1_X\backslash B$ and 1_X is the only $fg\alpha$-open set in (X,τ) containing $1_X\backslash B$ and so i is $fg^*\alpha$-continuous. Now $1_X\backslash B$ is fuzzy semiopen set in (X,τ_1) and $i^{-1}(1_X\backslash B) = 1_X\backslash B$ which is not fuzzy semiopen in (X,τ). Hence i is not $f\alpha$-irresolute.

Note 3.41. The following two examples show that fuzzy semi-continuity and $fg^*\alpha$-continuity are independent notions.

Example 3.42. *fuzzy semi-continuity $\nRightarrow$ $fg^*\alpha$-continuity*
Let $X=\{a,b\}$, $\tau=\{0_X,1_X,A\}$, $\tau_1=\{0_X,1_X,B\}$ where $A(a)=0.5, A(b)=0.4, B(a)=0.5, B(b)=0.5$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau)\to(X,\tau_1)$. Then fuzzy α-open sets in (X,τ) are $0_X,1_X,A$ and fuzzy α-closed sets in $(X,\tau)$are $0_X,1_X,1_X\backslash A$, fuzzy semiopen sets in (X,τ) are $0_X,1_X,A,V$ where $A\le V\le 1_X\backslash A$. $fg\alpha$-closed sets in (X,τ) are $0_X,1_X,U$, $1_X\backslash A$where $U\nleq A$, $fg\alpha$-open sets in (X,τ) are $0_X,1_X,A$, $1_X\backslash U$ where $1_X\backslash U \ngeq 1_X\backslash A$. Now $i^{-1}(B)=B$ which is fuzzy semiopen in (X,τ) and so i is fuzzy semi-continuous. Again, $1_X\backslash B$ is $fg\alpha$-open set such that $B = 1_X\backslash B \le 1_X\backslash B$. But $\alpha cl_\tau(1_X\backslash B) = \alpha cl_\tau B = 1_X\backslash A \nleq 1_X\backslash B$. Therefore, $1_X\backslash B$ is not $fg^*\alpha$-closed and so B is not $fg^*\alpha$-open in (X,τ) and hence i is not $fg^*\alpha$-continuous.

Example 3.43. *$fg^*\alpha$-continuity $\nRightarrow$ fuzzy semi-continuity*
Consider Example 3.40. Here B is fuzzy semiopen in (X,τ_1). But $i^{-1}(B) = B \notin$ FSO(X,τ). Therefore, i is $fg^*\alpha$-continuous but not fuzzy semi-continuous.

Remark 3.44. The following two examples show that fg-continuous function and $fg^*\alpha$-continuous function are independent notions.

Example 3.45. *fg-continuity$\nRightarrow$ $fg^*\alpha$-continuity*
Consider Example 3.16. Since 1_X is the only fuzzy open set such that $1_X\backslash A \le 1_X$. Then$cl_\tau(1_X\backslash A) = 1_X$ and so $1_X\backslash A$ is fg-closed in (X,τ) and so A is fg-open set in(X,τ). Hence i is fg-continuous though it is not $fg^*\alpha$-continuous.

Example 3.46. *$fg^*\alpha$-continuity$\nRightarrow$ fg-continuity*
Let $X=\{a,b\}$, $\tau=\{0_X,1_X,A\}$, $\tau_1=\{0_X,1_X,B\}$ where $A(a)=0.4, A(b)=0.6, B(a)=0.7, B(b)=0.6$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau)\to(X,\tau_1)$. Now $1_X\backslash B \in \tau_1^c$. Then $i^{-1}(1_X\backslash B) = 1_X\backslash B$. Now any $fg\alpha$-open set in (X,τ)other than 0_Xcontains$1_X\backslash B$ and $\alpha cl_\tau(1_X\backslash B) = 1_X\backslash B$ and hence i is $fg^*\alpha$-continuous. But $1_X\backslash B \le A$ and $cl_\tau(1_X\backslash B) = 1_X\backslash A \nleq A$ and so i is not fg-continuous.

Theorem 3.47. *A fuzzy function $f:(X,\tau)\to(Y,\tau_1)$ is $fg^*\alpha$-continuous iff the inverse image of every fuzzy closed set in $Y$is $fg^*\alpha$-closed in X.*

Proof. Let f be $fg^*\alpha$-continuousand $F \in \tau_1^c$. Then $1_X\backslash F \in \tau_1$. Since f is $fg^*\alpha$-continuous, $f^{-1}(1_X\backslash F) = 1_X\backslash f^{-1}(F)$ is $fg^*\alpha$-open in X. Hence $f^{-1}(F)$ is $fg^*\alpha$-closed in X.
Conversely, let us suppose that $f^{-1}(F)$be$fg^*\alpha$-closed in X for every fuzzy closed set Fin Y. Let $V \in \tau_1$. Then $1_X\backslash V \in \tau_1^c$. By assumption, $f^{-1}(1_Y\backslash V) = 1_X\backslash f^{-1}(V)$ is $fg^*\alpha$-closed in X and so $f^{-1}(V)$ is $fg^*\alpha$-open in X and hence f is $fg^*\alpha$-continuous.

Theorem 3.48. *Let $f:(X,\tau)\to(Y,\tau_1)$ be an $fg\alpha$-continuous, f-pre-α-closed function, then $f(A)$ is $f\alpha g$-closed in (Y,τ_1) for every $fg^*\alpha$-closed set Ain (X,τ).*

Proof. Let A be an $fg^*\alpha$-closed set in X and $V \in \tau_1$ be such that $f(A) \le V$. Then $A \le f^{-1}(V)$. As f is $fg\alpha$-continuous, $f^{-1}(V)$ is $fg\alpha$-open in (X,τ). Since A is $fg^*\alpha$-closed, and $A \le f^{-1}(V)$, $\alpha cl_\tau A \le f^{-1}(V) \Rightarrow f(\alpha cl_\tau A) \le ff^{-1}(V) \le V$. Since f is f-pre-α-closed, $f(\alpha cl_\tau A)$ is fuzzy α-closed in (Y,τ_1). Therefore, $\alpha cl_{\tau_1}\big(f(\alpha cl_\tau A)\big) = f(\alpha cl_\tau A) \le V$. Now, $A \le \alpha cl_\tau A \Rightarrow f(A) \le f(\alpha cl_\tau A) \Rightarrow \alpha cl_{\tau_1}\big(f(A)\big) \le \alpha cl_{\tau_1}\big(f(\alpha cl_\tau A)\big) = f(\alpha cl_\tau A) \le V$. Hence $f(A)$ is $f\alpha g$-closed in (Y,τ_1).

Theorem 3.49. *Let $f:(X,\tau)\to(Y,\tau_1)$ be fuzzy continuous, fuzzy pre-α-closed function, then $f(A)$ is $f\alpha g$-closed in (Y,τ_1) for every$fg^*\alpha$-closed set Ain (X,τ).*

Proof. Combining Theorem 3.2 and Theorem 3.8, we say that f is $fg\alpha$-continuous. Then by Theorem 3.48, $f(A)$ is $f\alpha g$-closed for every $fg^*\alpha$-closed set Ain X.

Theorem 3.50. *Let $f:(X,\tau)\to(Y,\tau_1)$ be an $fg\alpha$-continuous, f-*

pre-a-closed function and (Y,τ_1) is an $f\alpha T_b$-space, then $f(A)$ is $fg^*\alpha$-closed in (Y,τ_1) for every $fg^*\alpha$-closed set Ain (X,τ).

Proof. Let A be $fg^*\alpha$-closed in (X,τ) and V be any $fg\alpha$-open set in Y such that $f(A) \le V$. By Proposition 2.14, V is $f\alpha g$-open in Y. Since $(Y,\tau_1)$is $f\alpha T_b$-space, $1_X\backslash V$ being $f\alpha g$-closed in (Y,τ_1) is fuzzy closed in (Y,τ_1) and so V is fuzzy open in (Y,τ_1) . As f is$fg\alpha$-continuous, $f^{-1}(V)$ is $fg\alpha$-open in (X,τ). Since A is $fg^*\alpha$-closed in $(X,\tau)$and $A \le f^{-1}(V)$, $\alpha cl_\tau A \le f^{-1}(V) \Rightarrow f(\alpha cl_\tau A) \le ff^{-1}(V) \le V$. Since f is f-pre-α-closed, $f(\alpha cl_\tau A)$ is fuzzy α-closed in Y. Therefore, $\alpha cl_{\tau_1}\big(f(\alpha cl_\tau A)\big) = f(\alpha cl_\tau A) \le V$ and so $\alpha cl_{\tau_1}\big(f(A)\big) \le \alpha cl_{\tau_1}\big(f(\alpha cl_\tau A)\big) \le V$. Consequently, $f(A)$ is $fg^*\alpha$-closed in (Y,τ_1).

Remark 3.51. The composition of two $fg^*\alpha$-continuous functions need not be $fg^*\alpha$-continuous function as seen from the following example.

Example 3.52. Let $X=\{a,b\}$, $\tau=\{0_X,1_X,A\}$, $\tau'=\{0_X,1_X\}$, $\tau_1=\{0_X,1_X,B\}$ where $A(a)=0.5, A(b)=0.4, B(a)=0.6, B(b)=0.4$. Then (X,τ), (X,τ') and (X,τ_1) are fts's. Consider two identity functions$i:(X,\tau)\to(X,\tau')$and $i_1:(X,\tau')\to(X,\tau_1)$. Then clearly i and $i_1$are $fg^*\alpha$-continuous. But $i_1 o\, i:(X,\tau)\to(X,\tau_1)$ is not $fg^*\alpha$-continuous as seen from Example 3.10.

Theorem 3.53.*Let $f:(X,\tau)\to(Y,\tau_1)$ and $g:(Y,\tau_1)\to(Z,\tau_2)$ be two $fg^*\alpha$-continuous functions where $(Y,\tau_1)$is $fg^*\alpha T_c$-space. Then their composition $g\, o\, f:(X,\tau)\to(Z,\tau_2)$ is an $fg^*\alpha$-continuous function.*

Proof. Let $V\in\tau_2$. Then $g^{-1}(V)$ is $fg^*\alpha$-open in (Y,τ_1). As (Y,τ_1) is $fg^*\alpha T_c$-space, $1_Y\backslash g^{-1}(V)$ is fuzzy closed in (Y,τ_1) and so $g^{-1}(V)$ is fuzzy open in (Y,τ_1). Again, as f is $fg^*\alpha$-continuous, $f^{-1}(g^{-1}(V))$ is $fg^*\alpha$-open in (X,τ) and so $(gof)^{-1}(V)=f^{-1}(g^{-1}(V))$ for every $V\in\tau_2$. Consequently, $gof$is $fg^*\alpha$-continuous.

Theorem 3.54.*Let $f:(X,\tau)\to(Y,\tau_1)$ be an $f\alpha$-irresolute function and $g:(Y,\tau_1)\to(Z,\tau_2)$ be an $fg^*\alpha$-continuous function in (Y,τ_1) which is $fg^*\alpha T_\alpha$-space, then the composition $gof:(X,\tau)\to(Z,\tau_2)$ is $f\alpha$-continuous.*

Proof. Let $V\in\tau_2$. As g is $fg^*\alpha$-continuous, $g^{-1}(V)$ is $fg^*\alpha$-openin (Y,τ_1). Since (Y,τ_1) is $fg^*\alpha T_\alpha$-space, $1_X\backslash g^{-1}(V)$ is fuzzy α-closed in (Y,τ_1) and so ,$g^{-1}(V)$ is fuzzy α-open in (Y,τ_1). Since f is $f\alpha$-irresolute, $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)\in$ FαO(X). Hence gof is $f\alpha$-continuous.

Definition 3.55. For a fuzzy set$A$ in an fts (X,τ), $fg^*\alpha clA=\wedge\{B: A\le B, B$ is $fg^*\alpha$-closed in $(X,\tau)\}$.

Result 3.56. It is clear from Definition 3.56 that $fg^*\alpha clA = A$ for any $fg^*\alpha$-closed set A in an fts (X,τ).

Theorem 3.57.*Let $f:(X,\tau)\to(Y,\tau_1)$ be an $fg^*\alpha$-continuous function. Then for any$A\in I^X$, $f(fg^*\alpha cl_\tau A)\le cl_{\tau_1}f(A)$.*

Proof. Let $A\in I^X$. Then $cl_{\tau_1}f(A)\in\tau_1^c$ and as f is $fg^*\alpha$-continuous, $f^{-1}(cl_{\tau_1}f(A))$ is $fg^*\alpha$-closed in (X,τ). Hence by Result 3.57, $fg^*\alpha cl_\tau(f^{-1}(cl_{\tau_1}f(A)))=f^{-1}(cl_{\tau_1}f(A))$. Now $f(A)\le cl_{\tau_1}f(A)\Rightarrow A\le f^{-1}f(A)\le f^{-1}(cl_{\tau_1}f(A))$. Therefore, $f^{-1}(cl_{\tau_1}f(A))$ being a $fg^*\alpha$-closed set containing A. Then $fg^*\alpha cl_\tau A\le f^{-1}(cl_{\tau_1}f(A))$.Therefore, $f(fg^*\alpha cl_\tau A)\le cl_{\tau_1}f(A)$.

Corollary 3.58.*Let $f:(X,\tau)\to(Y,\tau_1)$ be a fuzzy continuous function. Then for any $A\in I^X$, $f(fg^*\alpha cl_\tau A)\le cl_{\tau_1}f(A)$.*

Proof. The proof follows from the fact that every fuzzy continuous function is $fg^*\alpha$-continuous and from Theorem 3.57.

4 $fg^*\alpha$-OPEN FUNCTIONS AND $fg^*\alpha$-CLOSED FUNCTIONS

In this section two new types of functions viz. $fg^*\alpha$-open function and $fg^*\alpha$-closed function have been introduced and studied and found the relationship of these two functions with fuzzy open function and fuzzy closed function.

Definition 4.1. A function$f:(X,\tau)\to(Y,\tau_1)$ is said to be $fg^*\alpha$-open function if the image of every fuzzy open set in (X,τ) is $fg^*\alpha$-open in (Y,τ_1).

Definition 4.2.A function$f:(X,\tau)\to(Y,\tau_1)$ is said to be $fg^*\alpha$-closed function if the image of every fuzzy closed set in (X,τ) is $fg^*\alpha$-closed in (Y,τ_1).

Theorem 4.3.*Every fuzzy open function is $fg^*\alpha$-open.*

Proof. Let $f:(X,\tau)\to(Y,\tau_1)$ be a fuzzy open function and $V\in\tau$. Then $f(V)$ is fuzzy open set in (Y,τ_1). By Proposition 2.5, $f(V)$ is $fg^*\alpha$-open in (Y,τ_1) and hence f is $fg^*\alpha$-open function.

Remark 4.4. The converse of the above theorem need not be true as seen from the following example.

Example 4.5.*$fg^*\alpha$-open function $\nRightarrow$ fuzzy open function*
Let $X=\{a,b\}$, $\tau=\{0_X,1_X,A\}$, $\tau_1=\{0_X,1_X,B\}$ where $A(a)=0.4, A(b)=0.6, B(a)=0.5, B(b)=0.7$. Then (X,τ) and (X,τ_1) are fts's. Consider the identity function $i:(X,\tau_1)\to(X,\tau)$. Then $i(B)=B$. We claim that B is $fg^*\alpha$-open in (X,τ).
Now $1-B(a)=0.5, 1-B(b)=0.3$. As in Example 3.20, $U\ge 1_X\backslash B$, for all $fg\alpha$-open sets U in (X,τ) and $\alpha cl_\tau(1_X\backslash B)=1_X\backslash B\le U$ and hence $1_X\backslash B$ is $fg^*\alpha$-closed in (X,τ) and so B is $fg^*\alpha$-open in (X,τ). Consequently, $i$is $fg^*\alpha$-open function. But $B\notin\tau$ and hence i is not fuzzy open function.

Theorem 4.6.*Every fuzzy closed function is $fg^*\alpha$-closed.*

Proof. Let $f:(X,\tau)\to(Y,\tau_1)$ be a fuzzy closed function and $V\in\tau^c$. Then $f(V)\in\tau_1^c$. By Proposition 2.5, $f(V)$ is $fg^*\alpha$-closed and hence f is $fg^*\alpha$-closed function.

Remark 4.7. The converse of the above theorem need not be true as seen from the following example.

Example 4.8. *$fg^*\alpha$-closedfunction $\nRightarrow$ fuzzy closed function*
Consider Example 4.5. Here $1_X\backslash B \in \tau_1^c$ and so $i(1_X\backslash B) = 1_X\backslash B$which is $fg^*\alpha$-closed in (X,τ) but is not fuzzy closed in (X,τ). Hence i is $fg^*\alpha$-closed function though it is not fuzzy closed function.

Theorem 4.9. *A function $f:(X,\tau)\to(Y,\tau_1)$ is $fg^*\alpha$-closed iff for each $B \in I^Y$ and for each $G \in \tau$with $f^{-1}(B) \le G$, there exists an $fg^*\alpha$-open set F in Y such that $B \le F$, $f^{-1}(F) \le G$.*

Proof. Let $B \in I^Y$ and $G \in \tau$be such that $f^{-1}(B) \le G$. Then $1_X\backslash G \in \tau^c$. As f is $fg^*\alpha$-closed function, $f(1_X\backslash G)$ is $fg^*\alpha$-closed in Y. Let $F = 1_Y\backslash f(1_X\backslash G)$. Then F is $fg^*\alpha$-open in Y. Now $1_X\backslash G \le 1_X\backslash f^{-1}(B) = f^{-1}(1_Y\backslash B)$. Therefore, $f(1_X\backslash G) \le ff^{-1}(1_Y\backslash B) \le 1_Y\backslash B$ and so $1_Y\backslash f(1_X\backslash G) \ge B \Rightarrow B \le F$and $f^{-1}(F) = f^{-1}\big(1_Y\backslash f(1_X\backslash G)\big) = 1_X\backslash f^{-1}f(1_X\backslash G) \Rightarrow 1_X\backslash G \le f^{-1}f(1_X\backslash G)$. Therefore, $\ge 1_X\backslash f^{-1}f(1_X\backslash G) = f^{-1}(F) \Rightarrow f^{-1}(F) \le G$.
Conversely, let $U \in \tau^c$. Then $1_X\backslash U \in \tau$. Now $f^{-1}\big(1_Y\backslash f(U)\big) = 1_X\backslash f^{-1}f(U)$. Since, $U \le f^{-1}f(U)$, $1_X\backslash f^{-1}f(U) \le 1_X\backslash U$. Therefore, $f^{-1}\big(1_Y\backslash f(U)\big) \le 1_X\backslash U$, where $1_Y\backslash f(U) \in I^Y$. Then there exists an $fg^*\alpha$-open set F in Y such that $1_Y\backslash f(U) \le F$and $f^{-1}(F) \le 1_X\backslash U$. Therefore, $U \le 1_X\backslash f^{-1}(F)$. Hence $1_Y\backslash F \le f(U) \le f(1_X\backslash f^{-1}(F)) \le 1_Y\backslash F \Rightarrow f(U) = 1_Y\backslash F$ and so $f(U)$ is $fg^*\alpha$-closed in Y. Consequently, f is $fg^*\alpha$-closed function.

Theorem 4.10. *The function $f:(X,\tau)\to(Y,\tau_1)$ is fuzzy closed function and $g:(Y,\tau_1)\to(Z,\tau_2)$ is$fg^*\alpha$-closed function, then their composition $g\, o\, f:(X,\ \tau)\to(Z,\tau_2)$ is $fg^*\alpha$-closed function.*

Proof. Let $G \in \tau^c$. Then as f is fuzzy closed function,$f(G) \in \tau_1^c$. As $g$is $fg^*\alpha$-closed function, $g\big(f(G)\big) = (gof)(G)$ is $fg^*\alpha$-closed in (Z,τ_2). Consequently, gof is $fg^*\alpha$-closed function.

Theorem 4.11. *Let $f:(X,\tau)\to(Y,\tau_1)$ and $g:(Y,\tau_1)\to(Z,\tau_2)$ be such that their composition $g\, o\, f:(X,\ \tau)\to(Z,\tau_2)$ is an $fg^*\alpha$-closed function. Then the following statements are true :*

- *(i) If f is fuzzy surjective continuous, then g is $fg^*\alpha$-closed function.*
- *(ii) If f is fuzzy surjective $fg\alpha$-continuous and (X,τ) is an $f\alpha T_b$-space, then g is $fg^*\alpha$-closed function.*
- *(iii) If g is $fg^*\alpha$-continuous and injective, then f is fuzzy closed function.*

Proof. (i) Let $V \in \tau_1^c$. Since f is fuzzy continuous, $f^{-1}(V) \in \tau^c$. Since gof is $fg^*\alpha$-closed function, $(gof)(f^{-1}(V))$ is $fg^*\alpha$-closed set in Z. As f is surjective, $(gof)(f^{-1}(V))$= $g\left(f\big(f^{-1}(V)\big)\right) = g(V)$, proving that g is $fg^*\alpha$-closed function.
(ii)Let$V \in \tau_1^c$. Since f is $fg\alpha$-continuous, $f^{-1}(V)$ is $fg\alpha$-closed in X. By Proposition 2.14, $f^{-1}(V)$ is $f\alpha g$-closed in X. As (X,τ) is an $f\alpha T_b$-space, ,$f^{-1}(V)$ is fuzzy closed in X. As gof is $fg^*\alpha$-closed function, $(gof)\big(f^{-1}(V)\big) = g(V)$ (as f is surjective) is $fg^*\alpha$-closed set in Z. Hence g is $fg^*\alpha$-closed function.
(iii)Let$V \in \tau^c$. Since gof is $fg^*\alpha$-closed function, $(gof)(V) = g(f(V))$ is $fg^*\alpha$-closed in Z. Since g is $fg^*\alpha$-continuous and injective, $g^{-1}(gof)(V) = g^{-1}g\big(f(V)\big) = f(V)$ is fuzzy closed in Y. Hence f is fuzzy closed function.

Theorem 4.12. *If $f:(X,\tau)\to(Y,\tau_1)$ is $fg^*\alpha$-closed function, then $fg^*\alpha cl_{\tau_1}\big(f(U)\big) \le f(cl_\tau(U))$, for every $U \in I^X$.*
Proof. Let $U \in I^X$. Then $cl_\tau U \in \tau^c$. Since f is $fg^*\alpha$-closed, $f(cl_\tau U)$ is $fg^*\alpha$-closed set in Y. As $U \le cl_\tau U$, $f(U) \le f(cl_\tau U)$, by Definition 3.55, $fg^*\alpha cl_{\tau_1}\big(f(U)\big) \le f(cl_\tau(U))$.

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